Declining Discount Rates in Singapore's Market for Privately Developed Apartments

Eric Fesselmeyer, Haoming Liu, and Alberto Salvo*

October 29, 2020

Summary

Singapore's market for new privately developed apartments exhibits wide quasi-experimental variation in ownership tenure. We develop an empirical model in which prices are decomposed into the utility of housing services and a factor that shifts with asset tenure and the discount rate schedule, which we discipline to vary smoothly over time. We estimate discount rates that decline over time and, to accommodate the observed price differences, fall to 0.5-1.5% p.a. by year 400. The finding that households making sizable transactions do not entirely discount benefits accruing centuries from today is relevant, with the appropriate risk adjustment, for evaluating climate-change investments.

Keywords: Social discount rate, declining discount rates, policy evaluation, long time horizon, climate change, real estate

JEL classification: D61, H43, Q51, R32

*Eric Fesselmeyer, efesselm@monmouth.edu, Department of Economics, Finance and Real Estate, Monmouth University, 400 Cedar Avenue West, Long Branch, NJ 07764, USA. Haoming Liu, ecsliuhm@nus.edu.sg, and Alberto Salvo, albertosalvo@nus.edu.sg: Department of Economics, National University of Singapore, 1 Arts Link, Singapore 117570. A former version of this research circulated with the title "How Do Households Discount Over Centuries? Evidence from Singapore's Private Housing Market." We thank audiences at the ASSA meetings, the East Asian Association of Environmental and Resource Economics congress, Economics of Low-Carbon Markets workshop, Hitotsubashi University, IZA, National ChengChi University, National University of Singapore, Santa Clara University, the China Meeting of the Econometric Society, the IAERE conference, the Singapore Economic Review conference and, in particular, Sumit Agarwal, Philippe Bracke, Maureen Cropper, Stefano Giglio, Christian Gollier, Adam Jaffe, Matteo Maggiori, Edward Pinchbeck, Ivan Png, and Johannes Stroebel for helpful comments.

Data and code availability statement: All data and code used to generate the results in this paper are available on Dataverse https://doi.org/10.7910/DVN/KUBAXT.

Conflict of interest statement: The authors declare no competing interests.

1 Introduction

Public policies generally have dynamic implications, so the choice of how to discount ensuing costs and benefits over time is critical. The relevant horizon may extend over decades, and even centuries into the future. In particular, economic analysis of climate change mitigation relies heavily on the assumed structure of discount rates (Cline, 1992; Nordhaus, 1994). Nordhaus (2007a,b) criticizes the use of a 1.4% p.a. consumption discount rate in the Stern Review (2007), which made distant climate damage loom large and called for immediate action (Stern and Taylor, 2007; Stern, 2013). Weitzman (2007a) notes that "what to do about global warming depends overwhelmingly on the imposed interest rate" (p.715). US government guidelines prescribe constant discount rates of 3% p.a. when evaluating regulations that affect households, and "sensitivity analysis using a lower but positive discount rate [in the presence of] important intergenerational benefits or costs" (OMB, 2003; Greenstone et al., 2011). Since a 3% rate values $1 one century from now at only 5 cents today, IWG (2010) cites "ethical objections that have been raised about rates of 3% or higher" (p.23). Drupp et al. (2018) find that academics "who place more emphasis on market-based rates of return recommend higher social discount rates" (p.17). Cropper et al. (2014) compare US rates that are flat over the investment horizon to declining discount rates (DDR) adopted by some European governments. NAS (2017) recommends that estimates of the social cost of carbon for US federal rulemaking "incorporate the relationship between discount rates and economic growth to help account for uncertainty surrounding discount rates over long time periods" (p.3).

Urged on by this climate policy debate, a growing body of theory addresses how consumption should be discounted over the long run. According to an expert panel (Arrow et al., 2012), "theory provides compelling arguments for a declining certainty-equivalent discount rate" (p.21), for instance, through uncertainty in future consumption growth and a precautionary motive in a Ramsey model.[1] In contrast, there is a dearth of evidence on how economic agents actually make trade-offs between today and the distant future. Empirical work is rare, because markets for assets or claims with long-run maturities are rarely observed. Groom et al. (2005) survey the DDR literature and write: "The difficulty in the long run is the absence of financial assets whose maturity

[1] Theory models serial correlation or uncertainty in the consumption growth rate (Gollier, 2002, 2008, 2014, 2016; Weitzman, 2007b) or in the discount rate (Weitzman, 1998, 2001; Gollier and Weitzman, 2010). Gollier (2010) models uncertainty in (economic) consumption and environmental quality—two goods of limited substitutability—and Hoel and Sterner (2007) make shifting relative prices explicit. Of relevance to the different rates in US guidelines, Li and Pizer (2019) consider a setting in which taxes shift the shadow price of capital (and there is no benefit uncertainty), with the social discount rate converging over long horizons to the consumption rate of interest.

extends to the horizon associated with... global warming. Government bonds, for example, do not extend beyond 40 years in general" (p.465). Another empirical challenge is how to apply time preferences inferred from prices in one imperfectly substitutable asset class, such as bonds or real estate, to another, such as climate-change mitigation assets, with different risk profiles and price paths (Sterner and Persson, 2008; Gollier, 2010; Weitzman, 2013; Gollier, 2016; Giglio et al., 2018).

A small empirical literature, starting with Fry and Mak (1984), estimates households' discount rates from price differences across residential assets of varying lease lengths. Fry and Mak (1984) estimated a high 11% p.a. discount rate from multi-decade housing contracts purchased by credit-constrained households in Hawaii. More recently, studies have examined housing markets in the UK and its former colonies Hong Kong and Singapore, where the right to the property is either held in perpetuity—a perpetual lease—or reverts to the lessor following initial horizons of 50 to 999 years. Intuitively, by comparing prices for two leases of different remaining length (and otherwise comparable properties), this literature infers whether households today value differential benefits that begin to accrue on the date the first lease expires.

Contributions to this literature include Wong et al. (2008), Bracke et al. (2018), Gautier and van Vuuren (2014), and Giglio et al. (2015). Wong et al. (2008) compare transaction prices between 99-year and 999-year leases in Hong Kong, inferring a discount rate of 4.3% p.a. The authors assume, rather than test, that the "999-year tenure is long enough to... be taken as a proxy for freehold interests" (p.287). Bracke et al. (2018), using flat sales in Central London, estimate discount rates that decline from 5-6% for nearly expired leases to close to 3% for leases with nearly one century remaining. Gautier and van Vuuren (2014) use land-lease contracts in Amsterdam, with initial duration of about 50 years, to estimate a quasi-hyperbolic discounting model. Giglio et al. (2015) examine sales of used and new houses and apartments in the UK and Singapore. As do Wong et al. (2008) for Hong Kong, they group properties of broadly similar remaining lease length into lease range bins, then use OLS regression with hedonic controls to estimate how transaction prices co-vary with dummy variables for the different bins. For example, UK lease bins are 80-99, 100-124, 125-149, 150-300, 700-999 year and perpetual leases. Based on their earlier work, Giglio et al. (2018) adopt a "discount rate for real estate cash flows 100 or more years in the future [of] about 2.6%" (p.3).

We differ from the literature by developing a tractable empirical model of residential property prices as the discounted value of a long-run stream of housing services, which we take to transaction data to directly estimate a discount rate schedule. We use a 20-year sample of fairly homogeneous new apartment transactions for Singapore and exploit the wide range in lease length, from perpetual to multi-century to multi-decade leases. We provide a direct empirical test of whether the discount rate declines over the very long run. Using nonlinear least squares, we fit smoothly varying parametric forms to the discount rate as a function of time. As with the use of polynomials in distributed lag models (Almon, 1965), the alternative parametric forms for the discount rate only discipline it to vary smoothly from one period to the next, yet the variation over time can be downward, upward or nonexistent. We also fit nonparametric structures in which the discount rate is a step function of time and some smoothness is imposed through a trend acceleration penalty as in the HP filter (Phillips and Jin, 2015). Besides focusing on new construction (of which Singapore has had a steady supply) and apartments (more heterogeneous houses comprise 6% of Singapore's residences), we correct for a rich and novel set of property characteristics. These characteristics include apartment story and sales/payments ahead of construction/delivery, which we find to shift the utility of housing services in densely urbanized, high-rise Singapore. All these aspects, including sample, empirical model, estimation approach, and results, allow us to build on previous work. In particular, Giglio et al.'s (2015) Singapore exercise does not control for apartment story and sales ahead of construction delivery, and includes used property and houses.

Compared to the literature that backs out discount rates from hedonic regressions of transaction prices on lease contract type and other property characteristics, the discount rates we estimate directly from the detailed new apartment sample not only vary smoothly but also tend to be lower, raising the value of payoffs in the far-distant future, for a similar asset horizon and risk. Specifying the discount rate either as a step function or as an exponential function of time, discount rates are estimated at about 3% (p.a.) up to year 100, and thereafter drop to 0.5% by year 400-500. For other mathematically different parametric forms—in which either the discount rate is a logarithmic function of time, or the logarithm of the discount rate is a linear function of the logarithm of time—estimated discount rates dip below 2% by year 100, and thereafter fall more gradually to about 1.5% by year 400-500. The latter specifications are closer to the 2.6% used by Giglio et al. (2018) for real estate cash flows beyond one century. Our alternative DDR schedules are intermediate to those simulated from different time-series models of long-term US government bond yields (Newell and Pizer, 2003; Groom et al., 2007; Freeman et al., 2015).

Our key result of DDR estimated directly from property transaction data is very robust to model specification and sample composition. Because discount rates fall sufficiently fast and sufficiently low, there is some evidence that new apartments on historical multi-century leases trade at a non-zero discount relative to property owned in perpetuity. The approach is simple, transparent, and thus appealing.[2]

Beyond real estate, van Binsbergen et al. (2012, 2014) document DDR in equity. These studies use data from derivatives markets (options, futures) to recover the prices of dividend strips at varying horizons up to a decade, and find that short-term dividends have a higher risk premium than long-term dividends. The empirical findings are consistent with the Lettau and Wachter (2007) model, whereby mean reversion in risky cash flows generates a downward-sloping term structure for the equity risk premium. van Binsbergen and Koijen (2017) review work on the term structure of equities and extend the evidence to other asset classes such as Treasuries and corporate bonds. Giglio et al. (2018) also use the Lettau and Wachter (2007) model to rationalize, through mean reversion in risky real-estate cash flows, DDR in real estate—a sector they argue faces "substantial exposure to both consumption risk and climate risk" (p.36).

In the remainder of the paper, Section 2 discusses the institutions and the data. Section 3 develops the empirical model and estimation. Section 4 reports estimates and Section 5 concludes, comparing our estimated discount rate schedules to schedules that are effective in policy today. All section, table, and figure numbers referenced below that are preceded by an "A," as in A.1, A.2, and so on, are contained in Fesselmeyer et al. (2020) as an Online Appendix, which is available on the Journal of Applied Econometrics website.

2 Institutional background and data

Choice of sample. Singapore's residential housing market consists of three types of properties: (i) apartments in high-rise buildings developed by a government agency, the Housing and Development Board (HDB); (ii) apartments in buildings, often high-rise, developed by private companies; and (iii) detached and semi-detached houses, which are also developed by private companies. New HDB apartments are purchased only by Singaporeans, and at subsidized prices. Privately developed apartments are purchased at market prices, mainly by Singaporeans but also by foreigners. Following local practice, we refer to these privately developed apartments—as opposed to HDB

[2] We model the discount rate in reduced form, abstracting away from uncertainty, but the approach is amenable to imposing restrictions derived from embedding discounting in a growth model that also models uncertainty.

apartments—as condominiums. Houses are sold to citizens and sometimes, with permission from the Singapore Land Authority, to permanent residents and foreigners.

There are 1.3 million housing units in Singapore. Home ownership among households headed by a citizen or permanent resident is a high 90% (Department of Statistics, 2015). In this nation of owner-occupiers, residential property is a widespread component of wealth, accounting for one-half of household net worth (Phang, 2001; Agarwal and Qian, 2017; Department of Statistics, 2018). Of these 1.3 million units, 75.1% are HDB apartments, 18.3% are privately developed (condominium) apartments, and 5.7% are houses (0.9% are unclassified). We examine condominiums, as their purchases are not subsidized or restricted (Appendix A.6). To control for aging, we also focus our sample on purchases of new, rather than new and used, construction. New detached and semi-detached houses are a small share of the market and may vary in unobservable ways. Our sample consists of new condominium purchases over the January 1995 to January 2015 period. It is for this relatively homogeneous sample that we exploit quasi-experimental variation in ownership tenure.

Singapore residential property ownership comprises "freeholds" and "leaseholds." Freeholds are assets owned in perpetuity, in contrast to leaseholds, in which the land and the housing infrastructure built on the leased land revert to the lessor—typically the state—when the lease expires. This land tenure system dates to the early 1800s, when Singapore was under British colonial rule (Lornie, 1921). Pursuant to the Letters Patent issued on November 27, 1826, the land title system under English law became the basis of Singapore's land law (Taylor Wessing, 2012). Much of the land that was developed over the 19th and much of the 20th centuries was in the form of perpetual leases and, in relatively smaller volume, leases with typical initial tenure of 999 years. The tenure remaining on these multi-century leases that survived to our 1995-2015 sample period ranges from 825 to 986 years. In practice, developers acquire rights to these lands (with no land rents payable subsequent to acquisition), build new condominiums on them, and sell them to households at their remaining tenure—say, 875 years. Our analysis characterizes each property by its exact remaining tenure from the moment housing services begin.

Following independence in 1965, the government embarked on a large-scale program to buy back some privately held land (Phang and Kim, 2011). The aim was to expand the housing stock and redevelop derelict areas. After the 1992 State Lands Act, a 99-year term became the norm for state-leased land, whether for new development or redevelopment, by the housing agency and private developers alike. The majority of condominiums built on land subsequently released by the government had a tenure of 99 years from the date a property developer acquired the rights. In contrast, the majority of condominiums built on privately owned land, whose titles were often issued by the British colonial government, were either perpetual or multi-century leases.

History has thus shaped a unique context in which new condominium projects under varying ownership tenure—ranging from perpetual to multi-century to multi-decade leases—are built by the same developers, in close proximity, both in space and in time. By "project," we refer to a collection of adjacent buildings or towers, each with many apartments that share a land parcel, name, and facilities such as a street entrance. Within a project, there is no variation in lease contract. For example, the Botannia, completed in 2009 on a 956-year lease from 1928, consists of 11 apartment towers. Next door to the Botannia, the 8-tower, aptly named Infiniti is a perpetual lease and was completed in 2008.

Hedonic analysis requires that the empiricist specify the granularity at which to control for spatial heterogeneity. Our specifications vary between 5-digit and 3-digit location controls, e.g., 12772x or 127xxx. With tight controls, 96% of 5-digit locations contain a single lease type—perpetual, multi-century, or multi-decade, e.g., location 12772x comprises only Botannia apartments and perfectly predicts their "956-years-from-1928" lease. Appendix Figure A.1(a) shows projects in the $100 - 96 = 4\%$ of 5-digit locations that contain more than one lease type. With less granular controls, 58% of 3-digit locations contain more than one lease type (Appendix Figure A.1(b)). In the example, location 127xxx encompasses both Botannia's multi-century lease and Infiniti's perpetual lease, thus preserving variation of interest. Fortunately, our finding of DDR is not sensitive to how we control for location. Appendix A.2 and Figure A.2 show that even 3-digit fixed effects control for location at quite a granular level. Because residences in the city-state agglomerate in 100 km^2 (14% of its land area), each 3-digit location corresponds, on average, to a square of side less than 1 km.

Figure 1 depicts the location of condominium projects in 5-year intervals. Each triangle, square, or circle marks a project on a perpetual, multi-century, or multi-decade lease, respectively, with new apartment sales recorded in a subperiod. The figure illustrates a key feature of the data: Whenever a new apartment under one lease type was sold, new apartments in neighboring projects and under other lease types tended to be sold. Local neighborhoods with new apartments on perpetual and multi-century leases, with their shared colonial history, have also offered new apartments on multi-decade leases, thanks to the government's land acquisition program. This enables us to identify the effect of tenure length on value—and thus the discount rate schedule—separately from the effect of location and time. The figure also shows that greenfield projects, on land assigned for residential development after the 1992 State Lands Act, mostly had initial tenure of 99 years; see the more scattered, peripheral parts of the city-state, such as the northeast, that house relatively more projects on multi-decade leases. For this reason, we complement our analysis of the full sample by considering a subsample of areas with availability of (at least) perpetual and multi-century leases; these residential areas tend to be more central and established.

Data. We extract private residential property transactions between January 1995 and January 2015 from the Urban Redevelopment Authority's Real Estate Information System (REALIS), which contains nearly the universe of new condominium transactions (Appendix A.1). According to this agency, the stock of condominium apartments grew from 53,429 in 1994 to 237,274 in 2014, i.e., an increase of 183,845 units. Reassuringly, REALIS contains 179,505 new sale records. Focusing on new property allows us to better control for unobserved quality. With used property, we would need to disentangle the price effects of highly correlated depreciation and remaining lease. The quality of used apartments traded with identical observed characteristics could differ considerably due to maintenance. In contrast, quality differences between new apartments—for instance, a perpetual lease and a neighboring 875-year lease—purchased at about the same time, are likely to be small.

We observe the date and price of the transaction, the year building construction was completed, the initial duration of land tenure, and the date on which tenure was originally granted. For example, one observation pertains to a condominium apartment purchased on 4/4/2008 for S$ 2,817,000 (Singapore dollars), with construction completed in 2007 and a lease of "998 Yrs From 12/27/1875." For this transaction, we compute the remaining tenure at the date of purchase to be $998 - (2008 - 1875) = 865$ years. We also observe the apartment's size in m^2 and address, from which we extract the story and the 6-digit postal code (that identifies the building), and the condominium project's name. Our sample consists of 179,218 units (99.9% of the 179,505 new sale records in REALIS), pertaining to 1,672 unique condominium projects.

We adjust nominal prices to account for variation in Singapore's Consumer Price Index (CPI), converting transaction prices to January 2014 Singapore dollars. Units are typically sold during construction. In the sample, 83% of transactions for perpetual and multi-century leases alike happen no later than the year before the apartment is delivered.[3] For such transactions, the median

[3]For example, 89% of apartments in the multi-century-lease Botannia and 98% of apartments in the perpetual-

time between purchase and delivery is 3 years, with buyers following a graduated payment schedule until construction is completed and the keys are handed over, at which time the remaining balance is due in full. We follow the typical payment schedule and use the CPI to compound (upward adjust) the prepaid components of the purchase price to the time construction is completed and the flow of housing services begins (Appendix A.3). Intuitively, making partial payment on an apartment years before it is delivered and housing benefits start accruing is akin to paying a higher price when the property is delivered. To control for unobserved heterogeneity (e.g., the best views "sell like hotcakes"), our analysis also corrects for the time between apartment purchase and construction completion, using 1-year bins from the negative to the positive domain. As expected, we find that prices are higher for apartments sold in the early phase of construction.

In the majority case in which apartments are purchased during construction, we take the time between construction completion (*not* sales) to lease expiry for the purpose of calculating the remaining asset life. For example, the 2010 purchase of a unit in the Interlace, on a "99 Yrs From 2/11/2009" lease, with construction completed in 2013, amounts to $99 - (2013 - 2009) = 95$ years of remaining lease length when housing benefits begin (not $99 - (2010 - 2009) = 98$ years). These data-handling procedures, such as correcting for the CPI and accounting for the fact that most new units are purchased ahead of delivery, should increase estimation precision.

Panel I of Table 1 summarizes the density of remaining lease against transaction year across the full sample of 179,218 purchases. We also summarize a subsample of 31,072 purchases that we use for sensitivity analysis: We restrict purchased apartments (of any lease type) to 3-digit areas in which at least perpetual and multi-century leases were traded. The ratio of multi-century to perpetual leases is higher in this subsample than in the full sample, as these areas are located in Singapore's more established neighborhoods (Figure 1). Over time, condominium sales have grown and the proportion of new units with 876 to 986 years remaining on the lease has fallen, while the proportion of leases with 825 to 875 remaining years has risen.

Panel II of Table 1 shows that mean apartment size and purchase price are, respectively, 108 m^2 and S$ 1.4 million, or about US$ 1 million. Across transactions, the number of apartments within the same project averages 370, and 70% of apartments are developed on large land parcels, according to an official designation. The average apartment is on the 9th story; the sample includes high-rises up to 70 stories. Our analysis allows these characteristics to shift the utility of housing

lease Infiniti were purchased before construction was completed. Appendix Table A.1 shows a similar pattern across lease types.

services. For example, high stories may provide better views or quieter environments; at the same time, a high story may be associated with a large land parcel or large project size, and more households in the condominium can potentially lead to management issues. Our analysis thus controls for apartment story, project size and large land parcel. The data further indicate that condominiums are developed quickly; for example, the median time to completion is 4 years.

Appendix Figure A.3 shows similar distributions by lease type over apartment size, distance to a mall (as a further proxy for localized neighborhood characteristics), building height, and buyer age, particularly for perpetual and multi-century leases. Building height distributions exhibit modes at 8 stories or so, and the distribution for multi-decade leases exhibits a second mode at 16 stories. Underscoring the longer lease types' proximity, given their shared colonial history, buyers of Chinese ethnicity accounted for 94% and 93%, and purchases with a mortgage accounted for 70% and 71%, respectively, of perpetual and multi-century leases in a 2000-2009 sample (He et al., 2020).

While a homeowner cannot expect (himself or his near descendants) to be alive to enjoy the stream of housing services produced by his residential asset one century (or a few centuries) in the future, Singapore has a very active resales market. For example, 40% of the new apartments sold in the first half of our sample period were resold within a decade, as homeowners trade up or liquidate their residential assets for cash. Relatedly, in a subsample of transactions for which buyer age is observed, it is not the case that older buyers choose shorter tenure rights (Appendix Figure A.3(d)). In robustness tests, we control for shared property amenities such as a swimming pool, and allow the discount rate schedule to vary according to buyer age where available (Appendix Tables A.3 and A.10).

Our assumption regarding respect for contracts—namely, that lessees enjoy the right to full term on their assets prior to the lessor's taking over—implies that the residual value of maturing properties is zero at the end of the land tenure.[4] Households pay a premium for longer leases because they generate a longer utility stream. We note that the expiration of the shorter leases in our sample is not imminent, and contract expiry is not salient to agents. The margin we examine is not that associated with longer leases vs. nearly expired leases, as found in Bracke et al. (2018).

[4]Unmodeled subsidized extensions (or benefits) on nearly expired contracts would bias estimated discount rates upward, making them conservative. The Singapore Land Authority states that general government policy is to allow leases to expire without extension. A developer who acquires aging property on ongoing yet unexpired 99-year leases, to be torn down and developed anew, can request from the government, for a fee, a "top up" to a 99-year lease.

3 Empirical model: Discounted value of housing services

We specify apartment i's time-invariant flow utility from housing services, u_i, as shifting with property characteristics, X_i, such as the size of the unit, the story it is on, and its detailed location:

$$u_i = u\left(X_i; \theta\right),$$

where θ is a row vector of parameters to be estimated. We lay out the empirical model in discrete time; thus u_i is the value of housing services per year, valued at the start of that year. Index the first year in which these benefits accrue to the buyer by $t = 1$. Housing benefits accrue over a finite or infinite lease of $L_i \in [2, \infty]$ years, and are modeled as certain. The other primitive in the model is the schedule of annual discount rates, $r_t > 0$, which can vary over time and on which we subsequently impose alternative structures. The sum of discounted value of the utility stream at $t = 1$—the moment that construction is completed and the buyer takes hold of the asset—is

$$V\left(X_i, L_i; \theta, \mathbf{r}\right) = u\left(X_i; \theta\right) \phi\left(L_i; \mathbf{r}\right) = u\left(X_i; \theta\right) \left(1 + \sum_{t=2}^{L_i \in [2,\infty]} \frac{1}{\prod_{s=1}^{s=t-1} (1 + r_s)}\right),$$

where $\mathbf{r} = (r_1, r_2, ...)$. Thus, for example, the present value of housing services in the first, second, and third periods are u_i, $u_i(1 + r_1)^{-1}$, and $u_i(1 + r_1)^{-1}(1 + r_2)^{-1}$, respectively (by present value we mean value at the start of $t = 1$). As modeled, discount rate r_t discounts benefits from year $t + 1$ to year t; it is a forward rate, as in Arrow et al. (2014).[5]

Identification follows from the fact that the value function $V\left(X_i, L_i; \theta, \mathbf{r}\right)$ factors into the flow utility of housing, which shifts with property characteristics, and a second factor that shifts with lease length and the discount rate schedule. We denote this second factor $\phi\left(L_i; \mathbf{r}\right)$. As the discounted sum of a stream of unitary flows, it can be interpreted as the asset's price multiple, i.e., a price-flow utility ratio.

We model the transaction price of the property, p_{im}, as the underlying value scaled by an exponential function of the sum of two unobservable shocks: a shock that varies across transactions but is common to the market m in which the transaction took place, denoted ξ_m; and an idiosyncratic

[5] Were housing services to decline with tenure—e.g., due to poor maintenance—this would conservatively overstate the implied discount rate, which we already find to be low. To see this, imagine two new properties, one with a 69-year lease, the other with 99 years (but otherwise identical), and assume that beginning in year 70, poor maintenance renders the second property unlivable. Both new properties would then transact at the same price (at $t = 1$), implying high discount rates out into the future in a model with time-invariant service flow such as ours.

mean-zero shock to the transaction of property i in market m, denoted ε_{im}:

$$p_{im} = V\left(X_i, L_i; \theta, \mathbf{r}\right) e^{\xi_m + \varepsilon_{im}}. \tag{1}$$

The market effect ξ_m may be due, for example, to the business cycle (the state of the economy) and to seasonality, to be captured by year fixed effects and quarter-of-year fixed effects, respectively. According to (1), prices are the discounted value of housing services, which are determined by property characteristics, and are subject to market shocks. The estimating equation is

$$\ln p_{im} = \ln u\left(X_i; \theta\right) + \ln \phi\left(L_i; \mathbf{r}\right) + \xi_m + \varepsilon_{im}. \tag{2}$$

In an alternative model, the relationship between the transaction price and underlying value is

$$p_{im} = V\left(X_i, L_i; \theta, \mathbf{r}\right) + \xi_m + \varepsilon_{im} = u\left(X_i; \theta\right) \phi\left(L_i; \mathbf{r}\right) + \xi_m + \varepsilon_{im}. \tag{3}$$

How $\phi\left(L_i; \mathbf{r}\right)$ varies across properties provides a measure of the transaction price variation in the data that is explained by differences in tenure valued from the present day. An apartment on a 875-year lease should roughly trade at a fraction $\phi\left(L_i = 875; \mathbf{r}\right) / \phi\left(L_i \to \infty; \mathbf{r}\right)$ of the price of a comparable apartment on a perpetual lease. A 94-year lease should trade at a $\phi\left(L_i = 94; \mathbf{r}\right) / \phi\left(L_i \to \infty; \mathbf{r}\right)$ fraction of a comparable perpetuity. Fixing property and market characteristics, X_i and ξ_m, it is this co-variation between tenure and prices that reveals how households discount many decades and centuries into the future.

3.1 Estimation algorithm

Let vector γ parameterize the discount rate schedule. We write $r_t = r\left(t; \gamma\right)$ and $\mathbf{r} = r\left(\gamma\right)$. Models (2) and (3) can be estimated by nonlinear least squares (NLLS). Specifically, we respectively solve

$$\underset{\gamma, \theta, \xi}{\operatorname{argmin}} \sum_{i=1}^{N} \left(\ln p_{im} - \ln u\left(X_i; \theta\right) - \ln \phi\left(L_i; \mathbf{r}\right) - \xi_m\right)^2, \tag{4}$$

$$\text{or, } \underset{\gamma, \theta, \xi}{\operatorname{argmin}} \sum_{i=1}^{N} \left(p_{im} - u\left(X_i; \theta\right) \phi\left(L_i; \mathbf{r}\right) - \xi_m\right)^2, \tag{5}$$

$$\text{subject to } \mathbf{r} = \left(r_1\left(\gamma\right), r_2\left(\gamma\right), ...\right) > 0.$$

where we collect all market fixed effects in a row vector $\xi = (\xi_m)$. The constrained optimization searches for parameters that minimize, across the N transacted properties in the sample, the sum of squared residuals (RSS). The key primitive of interest is the discount rate schedule, $\mathbf{r} = (r_1, r_2, ...)$, which we allow to vary over time either parametrically or nonparametrically. (For notational convenience, in the remainder of this section we omit the rate schedule parameters γ from $\mathbf{r}$.)

To reduce the computational burden, on implementing each model we further specify the flow utility of housing such that the estimating equation is linear in flow-utility parameters. Specifically, $u(X_i; \theta)$ is given by $e^{X_i \theta}$ in model (2) where log price is the dependent variable, and by $X_i \theta$ in model (3) where price is the dependent variable. Fixing $\mathbf{r}$, each estimation equation is now linear in the remaining parameters, θ and ξ_m, which need not be included in the nonlinear search.

For example, consider model (3). Express the scalar ξ_m as $\xi D_i'$, where D_i is a row vector of market dummies for property i, stack all observations, and use matrix notation to write[6]

$$p = \begin{bmatrix} \phi(L; \mathbf{r}) X & D \end{bmatrix} \begin{bmatrix} \theta & \xi \end{bmatrix}' + \varepsilon = Z(\mathbf{r}) \alpha + \varepsilon,$$

where $Z(\mathbf{r}) := \begin{bmatrix} \phi(L; \mathbf{r}) X & D \end{bmatrix}$ and vector $\alpha = \begin{bmatrix} \theta & \xi \end{bmatrix}'$ contains flow-utility parameters and property-market shocks. The RSS function to be minimized is then

$$(p - Z(\mathbf{r}) \alpha)' (p - Z(\mathbf{r}) \alpha).$$

The first-order condition with respect to α is linear in α, so during estimation the θ and ξ parameters can be concentrated out, i.e., $\alpha(\mathbf{r}) = \left(Z(\mathbf{r})' Z(\mathbf{r})\right)^{-1} Z(\mathbf{r})' p$. The optimization routine then searches only over the rate schedule parameters γ:

$$\underset{\gamma}{\operatorname{argmin}} \, (p - Z(\mathbf{r}(\gamma)) \alpha(\mathbf{r}(\gamma)))' (p - Z(\mathbf{r}(\gamma)) \alpha(\mathbf{r}(\gamma))).$$

A similar procedure applies to our implementation of model (2) with $u(X_i; \theta) = e^{X_i \theta}$. Fixing $\mathbf{r}$, $\ln p_{im} - \ln \phi(L_i; \mathbf{r})$ is linear in θ and ξ_m.

[6] We abuse notation by writing $\phi(L; \mathbf{r}) X$, meaning that we multiply every $N \times 1$ column in X by the $N \times 1$ column vector $\phi(L; \mathbf{r})$, element by element.

3.2 Incidental parameters problem

Estimation of a nonlinear panel data model that includes fixed effects is subject to the incidental parameters problem (Arellano and Bonhomme, 2011). Regarding our models with location fixed effects (in X) and year and quarter fixed effects (in ξ), one may ask whether there are enough observations per fixed effect, particularly per 5-digit location, to render any bias small.

Several papers develop jackknife methods to mitigate incidental parameters bias when estimating nonlinear panel data models. A common scenario is when, relative to the number of individual fixed effects J in the cross-section dimension, the time dimension T is small. In the 1995-2015 sample, J/T averages 13 with 1516 narrow 5-digit fixed effects (i.e., 1516/(179,218/1516)) and a more favorable 0.2 with 187 less narrow 3-digit fixed effects. Hahn and Newey (2004) use the variation in the fixed-effects estimator as each time period is dropped (and then replaced), one at a time, to compare with the estimate from the entire sample, thus estimating and adjusting for any bias. Dhaene and Jochmans (2015) propose a jackknife adjustment that uses variation over subpanels of consecutive observations and allows for dependence in the time dimension. Fernandez-Val and Weidner (2016) consider models that include both cross-section fixed effects and time fixed effects. They modify Dhaene and Jochmans (2015), splitting data into subpanels in the cross-sectional dimension and in the time dimension, to minimize the bias caused by estimating both individual fixed effects and time fixed effects.

Following Fernandez-Val and Weidner (2016), let $\hat{\gamma}_{JT}$ be the fixed-effects estimator and define cross-sectional indices A and time-series indices B. Let $\tilde{\gamma}_{J,T/2}$ be the average of the 2-split jackknife estimators in the subpanels with $A = \{1, 2, \ldots, J\}$, and $B_1 = \{1, 2, \ldots, T/2\}$ or $B_2 = \{T/2+1, T/2+2, \ldots, T\}$. Let $\tilde{\gamma}_{J/2,T}$ be the average of the 2-split jackknife estimators in the subpanels with $B = \{1, 2, \ldots, T\}$, and $A_1 = \{1, 2, \ldots, J/2\}$ or $A_2 = \{J/2+1, J/2+2, \ldots, J\}$.[7] The bias-corrected estimator that we implement is then $\tilde{\gamma}^{bc}_{JT} = 3\hat{\gamma}_{JT} - \tilde{\gamma}_{J,T/2} - \tilde{\gamma}_{J/2,T}$.

3.3 Penalized nonlinear least squares

The empirical model can be implemented with different forms for the discount rate schedule; for example, by restricting the discount rate to be a step function of time. In this case, to allow multiple steps in the rate schedule, we can impose some smoothness on the objective function.

[7] We randomly allocate an equal number of locations to A_1 and A_2, and repeat 10 times, yielding 20 subpanels. Thus $\tilde{\gamma}_{J/2,T}$ is an average over 20 estimates.

Similar in spirit to the Hodrick-Prescott (HP) filter (Phillips and Jin, 2015), we impose smoothness by penalizing second-order differences (i.e., changes in the change in discount rates). Under model (2), optimization problem (4) is augmented by a rate acceleration (or deceleration) penalty:

$$\underset{\gamma,\theta,\xi}{\operatorname{argmin}} \sum_{i=1}^{N} \left(\ln p_{im} - \ln u\left(X_i;\theta\right) - \ln \phi\left(L_i;\mathbf{r}\right) - \xi_m\right)^2 + \lambda \sum_t \left(\left(r_{t+2} - r_{t+1}\right) - \left(r_{t+1} - r_t\right)\right)^2, \quad (6)$$

where λ is a smoothing (or tuning) parameter and, again, discount rate r_t discounts benefits from year $t+1$ to year t. We fix weight λ by first solving (4) (with $\lambda = 0$) to find the no-penalty RSS, then solve the augmented problem (6) setting λ at twice this RSS. In practice, the sum of squared second differences in rates is a small number, so the penalty term is small relative to the first term (the RSS) and but sufficient to impose some discipline on the nonparametric rate structure.[8]

4 Results

Discount rate as a smooth function of time. Table 2 shows results when we allow the discount rate schedule to vary smoothly over time. We fit three different parametric forms for $\mathbf{r} = r(\gamma)$, such as an exponential function of time:

$$r_t = r(t;\gamma) = \begin{cases} \max\left(\gamma_1 \exp\left(\gamma_2 (t-1)\right), \gamma_3\right) & \text{for } 1 < t \leq 10^6 \\ \max\left(\gamma_1 \exp\left(\gamma_2 \left(10^6 - 1\right)\right), \gamma_3\right) & \text{for } t > 10^6 \end{cases} \qquad \text{(exponential)}$$

Parameter $\gamma_1 > 0$ corresponds to the discount rate in period 1, i.e., r_1. We fix the year-1 rate at 4% p.a., and subsequently show sensitivity to estimating this parameter, or to fixing it at levels other than 4% p.a. In simulations of estimated time-series models of US government bond yields, Newell and Pizer (2003), Groom et al. (2007), and Freeman et al. (2015) fix the starting rate at 4% p.a.—i.e., the pattern of decline is estimated, but not the starting point.

Parameter $\gamma_2 \gtrless 0$ defines the slope of the rate schedule, i.e., declining if negative, rising if positive, or flat (constant discount rate). γ_2 is the key parameter of interest. We fix parameter $\gamma_3 > 0$, which sets a floor to the discount rate. Specifically, we impose the regularity condition that discount rates are bounded from below at $\gamma_3 = 0.01\%$ p.a., and provide some sensitivity analysis around this normalization. As the discount rate approaches 0, the value of an infinite utility stream

[8]The weighting—how much "smoothness" is desired—is rather arbitrary. We report the components of the optimized objective function and conduct robustness tests.

increases arbitrarily. Finally, to make estimation computationally tractable, we impose a flat rate schedule beyond year 1,000,000, restricting r_t for $t > 10^6$ to be equal to the estimated discount rate for year 1,000,000. Appendix A.4 provides expressions for the other two functional forms of time—logarithmic and hyperbolic—for which parameters $(\gamma_1, \gamma_2, \gamma_3)$ have the same interpretation.

Besides reporting on different forms for $\mathbf{r}$ (which enters ϕ), Table 2 specifies alternative location fixed effects (which enter u through X) and implements model variants (2) and (3) (which relate observables and unobservables). For all combinations of rate schedule form, location controls, and model, we obtain discount rates that decline over time. The rate decline in time is slightly steeper (γ_2 slightly more negative) with 5-digit than with 3-digit locations. Compared with model (3) in the bottom panel of Table 2, the cross-validation Mean Squared Error is lower for model (2) in the top panel. This suggests that the model with log price as the dependent variable and $u(X_i; \theta)$ given by $e^{X_i\theta}$ provides a better fit. With 5-digit controls, we are able to reject equality in favor of the model with log price as the dependent variable (Appendix Table A.4, panel I)—Appendix A.4 details the cross-validation procedure. Unsurprisingly, the Mean Squared Error is lower with more granular 5-digit than with 3-digit fixed effects.

Across the three functional forms for $\mathbf{r}$, shown in different columns of Table 2, the Mean Squared Error is similar. With 5-digit fixed effects (and the log price model), the square root of these Mean Squared Errors across parametric structures are within 1 S\$/m^2 of one another, yet we are still able to reject equality in favor of the logarithmic form (Appendix Table A.4, panel II). With 3-digit fixed effects, the exponential r_t exhibits statistically significantly lower Mean Squared Error, but the square root is within 4 S\$/m^2—a limited difference—of that of the other parametric structures. Even with fewer transactions per 5-digit location (a higher J/T), bias-corrected jackknife estimates $\tilde{\gamma}_{JT}^{bc}$ suggest that any incidental parameters bias remains small (Table 2, row labeled "Jackknife estimator").

To summarize Table 2, a cross-validation exercise suggests that model (2) (with log price as the dependent variable) outperforms model (3), though the decline in discount rates (slope γ_2) is similar across the two models. The decline in discount rates is similar irrespective of the granularity of location controls. The choice of functional form for the rate schedule matters, as Figure 2 illustrates (and Appendix Table A.4 reports), yet the schedule is firmly declining and the Mean Squared Error varies little. As a result of discount rates falling to 0.5% by year 400-500, the fitted exponential

r_t predicts a 6% price discount for multi-century leases relative to comparable perpetual leases.[9] Consistent with this finding, OLS regressions of apartment prices on coarse bins defined by lease length (and controls including 3-digit location) indicate that multi-century leases trade at a 4-6% discount relative to comparable perpetual leases (Appendix Tables A.2 and A.3).

Figure 3 shows what happens when we no longer fix the year-1 rate at 4% p.a. but rather jointly estimate this parameter (subject to the constraint that it lies between 0.01% and 10% p.a.) and the slope parameter γ_2. $\hat{\gamma}_1$ ranges between 2.5 and 8.4% p.a., depending mostly on the functional form for r_t, but the result that discount rates decline over time is robust, and the Mean Squared Error is similar (Appendix Table A.5). With $\hat{\gamma}_1 = 6.5\%$ p.a., rates under the logarithmic r_t (and 3-digit controls) fall sufficiently by year 500 to predict a 3% price difference between multi-century and perpetual leases. Appendix Figure A.5 fixes γ_1 at levels other than 4% p.a. As the short-run discount rate decreases from 6% to 3% p.a., the slope of the rate schedule flattens to compensate. Under exponential r_t, rates change less over the first few decades, to fall sharply thereafter. Rates under hyperbolic r_t decline sharply in the first decade but remain above 1% p.a. one millennium into the future. Rates under logarithmic r_t display an intermediate pattern to that of the other structures.

Appendix Figure A.6 illustrates the sensitivity with regard to the lower-bound rate. We plot fitted exponential r_t only, since estimates for the other structures are not sensitive to varying the lower bound γ_3 in the considered range. Even for the exponential, the differences are not large as we alternatively fix γ_3 at 0.1%, 0.01%, or 0.001% p.a.

One question that may arise is whether there is disutility from time-limited home ownership per se. The sensitivity analysis reported in Appendix Table A.7 includes an indicator for a perpetual lease in vector X_i, allowing for a differential flow utility of housing services when the asset is "owned forever." Identification follows from how long-run multi-century leases are valued relative to the multi-decade ones, and how these maturities are valued relative to perpetuities. Our DDR result is robust, with similar slopes relative to Table 2, and lower (but still high) precision. As we vary the model and the functional form for the rate schedule, $\hat{\theta}_{forever}$, the "asset owned forever" shifter, varies in sign and is mostly insignificant at conventional levels. This is consistent with discount rates declining smoothly over the lease length range, rather than any disutility from time-limited

[9] See the row labeled "avg $\hat{\phi}_i$ 825-986y/avg $\hat{\phi}_i$ perpetual" in the top panel of Table 2, columns 1a-2a showing 0.94. Appendix Table A.6 and Figure A.4 show that DDR again obtain—and the schedules are somewhat steeper—in the subsample of transactions in more established areas that are home to both perpetual and multi-century leases.

home ownership being significant.

Discount rate restricted to be flat over time. Current policy analysis in different countries adopts a constant discount rate (US), or a discount rate that is a step function of time (UK, France). Table 3, columns 1-2 provide a constant discount rate benchmark: On specifying $r_t = r = \gamma$ (one parameter), we obtain 2.1% p.a. with 5-digit and 2.2% p.a. with 3-digit location controls. Similar to above, estimated rates come out slightly lower with 5-digit fixed effects, and from here we focus on model variant (3), with log price as the dependent variable.

The remaining columns of Table 3 provide empirical motivation for time-varying discount rates. Here we let the discount rate be a step function of time, starting with a single jump at 100 years in columns 3-4, or at 800 in columns 5-6. Formally, we specify

$$r_t = r\left(t;\gamma\right) = \begin{cases} \gamma_1 \text{ for } 1 < t < t_1 \\ \gamma_2 \text{ for } t \geq t_1 \end{cases}$$

with the single cutoff at $t_1 = 100$ or $t_1 = 800$. In all cases, point estimates indicate DDR, $\hat{\gamma}_2 < \hat{\gamma}_1$. For the jump-at-year-800 form, the estimated jump is sufficiently large and precise that we are able to reject at the 1% significance level the hypothesis that γ_1 (2.1-2.4% p.a.) and γ_2 (0.0-0.6% p.a.) are equal. In column 6, with γ_2 falling to close to zero, the price-flow utility ratio for 825-986 year leases is 4% lower than for a comparable perpetual lease.

Table 4 reports on estimated forms for the rate schedule when we allow for multiple steps in r_t, namely ten or five steps one century wide, and impose some smoothness through an HP-like filter (objective function (6) and Appendix A.4). With 5-digit location controls, we obtain a discount rate of 2.5% p.a. in the first century, falling to 0.5% by year 500. With 3-digit controls, the discount rate starts a little higher, at 3.2-3.5% p.a. in the first century, and the decline is steeper. Figure 4 plots the fitted step functions of Table 4. Appendix Table A.11 and Figure A.7 show that our findings are fairly robust to changes in the specified smoothing parameter: doubling (resp., halving) λ slightly compresses (resp., decompresses) the discount rate schedule, and the cross-validation Mean Squared Error slightly increases. Table 4 indicates that, at the fitted parameters, the rate deceleration penalty is a small number relative to the RSS. A cross-validation exercise suggests that the 5-step rate schedule of Table 4 outperforms many but not all of the parametric forms of Table 2 (Appendix Table A.4, panel III).

By way of a summary, Figure 5(a) jointly plots the three smooth r_t schedules of Table 2 and the 5-step schedule of Table 4 (5-digit fixed effects). The step function tracks the logarithmic and hyperbolic r_t in years 1 to 200 and the exponential r_t in years 201 to 400, which may suggest that the parametric structures are too restrictive for the entire horizon. We return to Figure 5 in the Discussion. Finally, while our analysis has focused on time preferences, estimates on housing utility shifters are intuitive and reflective of Singapore's market. For example, a 1% increase in size is associated with an apartment price increase of 0.9%; price increases with story; and apartments purchased in the early phase of construction command higher prices—e.g., they have the best views. Overall, our key result that discount rates decline over time is robust to the structure imposed on the discount rate schedule, model specification, and location fixed effects.

5 Conclusion and policy implications

We summarize our contributions and briefly discuss their relevance, in particular, to policy on climate change. The empirical model we develop partials out from transaction prices, in a tractable yet theoretically appealing way, property characteristics such as location and time-varying market conditions. We use a wide range of lease lengths in a 20-year sample of new Singaporean apartment purchases to estimate a nonlinear discount rate schedule directly, which is a distinctive feature of our work. Singapore's local neighborhoods have offered new apartments on perpetual and multi-century leases with shared colonial history—as well as on multi-decade leases—at similar points in time, allowing us to separate out the effects of location and time. We amass a rich set of property characteristics, some of which were not used in previous work on discounting in this leading Asian city-nation's residential property markets. These include building story—which tends to be high relative to the US and Europe—and the time difference between purchase and construction completion, which correlates with unobserved quality. Recognizing important drivers of value such as these affords us estimation precision.[10]

We provide compelling evidence that in these fairly homogeneous private-housing markets, discount rates decline over long horizons. Our key DDR result is very robust to model specification, including the granularity of location controls, ranging from very tight 5-digit postal codes (1,516

[10]Future studies can collect data on buyer characteristics to study heterogeneity in discounting (Appendix Table A.10), thus contributing to a literature on individual (or personal) discount rates (Frederick et al., 2002; Warner and Pleeter, 2001). This literature has used individual choices in real-world markets, typically over shorter horizons than ours, to examine mechanisms and applications such as liquidity constraints and lifecycle consumption (Zeldes, 1989; Gourinchas and Parker, 2002), the value of a statistical life (Moore and Viscusi, 1989, 1990; Viscusi and Moore, 1989), and the energy efficiency gap (Hausman, 1979; Train, 1985; Gillingham and Palmer, 2014).

locations over a 100 km^2 residential area, or 0.07 km^2 per fixed effect) to less granular, but still quite fine, 3-digit controls (0.53 km^2 per fixed effect). We discipline rates to vary smoothly over long benefit horizons according to alternative parametric and nonparametric structures. While slopes differ somewhat in magnitude and rate of change, discount rates decline in all cases and the schedules are overlaid on one another. This is a very robust feature in our work. Importantly, the sign of slopes is not imposed; this can be positive or zero, and negative slopes are estimated as a test of DDR. Because discount rates fall sufficiently fast and sufficiently low for some specifications, there is some evidence that new apartments on historical multi-century leases trade at a non-zero discount relative to property owned in perpetuity.

Figure 5(b) compares our alternative fitted structures for the discount rate schedule to schedules that guide public policy in the UK, including the standard rates and reduced rates recommended for sensitivity analysis (HM Treasury, 2003, 2008), and the 2005 and revised 2013 rates used in France (Lebègue, 2005; Quinet, 2013). To emphasize, the plotted discount rate r_t discounts benefits from year $t + 1$ back to year t—i.e., the forward rate (not the "effective term structure," which is the rate that would discount benefits from year t back to year 0). The fitted exponential r_t tracks the UK schedule until the UK schedule levels off beyond year 300 at 1% p.a., while the schedule we estimate continues to decline through 0.5% p.a. over subsequent centuries. The fitted step function also declines to 0.5% p.a. by year 400. The fitted logarithmic and hyperbolic structures drop fast early on, in line with the French schedule, then level off from year 200 at about 1-1.5% p.a., above the fitted exponential and nonparametric r_t. Our result provides empirical support to governments that adopt DDRs to evaluate public policies that yield benefits over very long horizons. Bracke et al. (2018) argue that better understanding of housing market discount rates can be useful also in long-horizon policy settings including pension financing and infrastructure investments, which have features in common with housing such as low liquidity and location specificity.

Figure 5(c) compares our estimated schedules against DDRs simulated by Newell and Pizer (2003) and Groom et al. (2007), based on fitting alternative reduced-form time-series models to historical interest rates for long-term US government bonds. The DDR schedules proposed by studies in this empirical "Expected Net Present Value" literature follow from the serial correlation in government bond yield uncertainty (Weitzman, 1998; Arrow et al., 2014). The DDRs we estimate from Singapore residential property prices are, beyond the first century, intermediate to Newell and Pizer's random walk model and subsequent studies that used more flexible econometric models, e.g., Groom et al.'s state space model.[11] The fitted logarithmic and hyperbolic r_t track the Groom et al. schedule over the first four centuries.

In a Policy Forum for *Science*, Arrow et al. (2013) compare a constant 4% p.a. to the DDR schedules in Newell and Pizer (2003), Groom et al. (2007), and Freeman et al. (2015). Arrow and his 12 co-authors state: "In these studies, estimates of the social cost of carbon are increased by as much as two- to threefold by using a DDR, compared with using a constant discount rate of 4%, the historic mean return on U.S. Treasury bonds" (p.350). Gollier (2010) shows that the intertemporal pricing of two asset classes—one paying out monetary benefits and the other environmental amenities—depends on their substitutability and on the uncertainty that surrounds their future growth rates, which offers arguments in favor of an "ecological" discount rate that is smaller than the economic discount rate. Even so, these unknown preference and risk factors aside, Figure 5 suggests that the DDR we directly estimate from households' observed choices in a real-world market for new property would similarly raise the social cost of carbon substantially.

References

Agarwal, S. and Qian, W. (2017). Access to home equity and consumption: Evidence from a policy experiment. *Review of Economics and Statistics*, 99(1), pp. 40–52.

Almon, S. (1965). The distributed lag between capital appropriations and expenditures. *Econometrica*, 33(1), pp. 178–196.

Arellano, M. and Bonhomme, S. (2011). Nonlinear panel data analysis. *Annual Review of Economics*, 3, pp. 395–424.

Arrow, K. J., Cropper, M. L., Gollier, C., Groom, B., Heal, G. M., Newell, R. G., Nordhaus, W. D., Pindyck, R. S., Pizer, W. A., Portney, P. R., Sterner, T., Tol, R. S. J., and Weitzman, M. L. (2012). How should benefits and costs be discounted in an intergenerational context? *Resources for the Future Discussion Paper 12-53.*

Arrow, K. J., Cropper, M. L., Gollier, C., Groom, B., Heal, G. M., Newell, R. G., Nordhaus, W. D., Pindyck, R. S., Pizer, W. A., Portney, P. R., Sterner, T., Tol, R. S. J., and Weitzman, M. L. (2013). Determining benefits and costs for future generations. *Science*, 341(6144), pp. 349–350.

[11] Newell and Pizer (2003) fit an AR(p) model to the log of annual US interest rates (partly adjusted for CPI variation), with the sum of autoregressive parameters restricted to 1, i.e., an AR random walk model. They then use the estimated model to simulate thousands of interest rate paths. Following Weitzman (1998), the certainty-equivalent forward rate, r_t, is then given by $(1+r_t)^{-1} = E\left[e^{-r_1}e^{-r_2}...e^{-r_t}e^{-r_{t+1}}\right]/E\left[e^{-r_1}e^{-r_2}...e^{-r_t}\right]$, where the expectation is taken over the simulated paths. Groom et al. (2007) allow for time-dependent parameters by modeling an AR(1) process with an AR(p) coefficient. Freeman et al. (2015) use a more complete inflation history to model the process that drives the CPI separately from the one that generates the nominal interest rate.

Arrow, K. J., Cropper, M. L., Gollier, C., Groom, B., Heal, G. M., Newell, R. G., Nordhaus, W. D., Pindyck, R. S., Pizer, W. A., Portney, P. R., Sterner, T., Tol, R. S. J., and Weitzman, M. L. (2014). Should governments use a declining discount rate in project analysis? *Review of Environmental Economics and Policy*, 8(2), pp. 145–163.

Bracke, P., Pinchbeck, E. W., and Wyatt, J. (2018). The time value of housing: Historical evidence on discount rates. *Economic Journal*, 128(613), pp. 1820–1843.

Cline, W. R. (1992). *The Economics of Global Warming*. Institute for International Economics, Washington, D.C.

Cropper, M. L., Freeman, M. C., Groom, B., and Pizer, W. A. (2014). Declining discount rates. *American Economic Review*, 104(5), pp. 538–543.

Department of Statistics. (2015). *Singapore in Figures*. Singapore Department of Statistics.

Department of Statistics. (2018). *Household Sector Balance Sheet (M700981)*. Singapore Department of Statistics.

Dhaene, G. and Jochmans, K. (2015). Split-panel jackknife estimation of fixed-effect models. *Review of Economic Studies*, 82(3), pp. 991–1030.

Drupp, M., Freeman, M., Groom, B., and Nesje, F. (2018). Discounting disentangled. *American Economic Journal: Economic Policy*, 10(4), pp. 109–134.

Fesselmeyer, E., Liu, H., and Salvo, A. (2020). Online Appendix to Declining discount rates in Singapore's market for privately developed apartments.

Fernandez-Val, I. and Weidner, M. (2016). Individual and time effects in nonlinear panel models with large N, T. *Journal of Econometrics*, 192(1), pp. 291–312.

Frederick, S., Loewenstein, G., and O'Donoghue, T. (2002). Time discounting and time preference: A critical review. *Journal of Economic Literature*, 40(2), pp. 351–401.

Freeman, M. C., Groom, B., Panopoulou, E., and Pantelidis, T. (2015). Declining discount rates and the Fisher Effect: Inflated past, discounted future? *Journal of Environmental Economics and Management*, 73, pp. 32–49.

Fry, M. J. and Mak, J. (1984). Is land leasing a solution to unaffordable housing? *Economic Inquiry*, XXII, pp. 529–549.

Gautier, P. A. and van Vuuren, A. (2014). The estimation of present bias and time preferences using land-lease contracts. Mimeo, VU University of Amsterdam.

Giglio, S., Maggiori, M., and Stroebel, J. (2015). Very long-run discount rates. *Quarterly Journal of Economics*, 130(1), pp. 1–53.

Giglio, S., Maggiori, M., Rao, K., Stroebel, J., and Weber, A. (2018). Climate change and long-run discount rates: Evidence from real estate. Mimeo, University of Chicago.

Gillingham, K. and Palmer, K. (2014). Bridging the energy efficiency gap: Policy insights from economic theory and empirical evidence. *Review of Environmental Economics and Policy*, 8(1), pp. 18–38.

Gollier, C. (2002). Discounting an uncertain future. *Journal of Public Economics*, 85(2), pp. 149–166.

Gollier, C. (2008). Discounting with fat-tailed economic growth. *Journal of Risk and Uncertainty*, 37(2), pp. 171–186.

Gollier, C. (2010). Ecological discounting. *Journal of Economic Theory*, 145(2), pp. 812–829.

Gollier, C. (2014). Discounting and growth. *American Economic Review*, 104(5), pp. 534–537.

Gollier, C. (2016). Evaluation of long-dated assets: The role of parameter uncertainty. *Journal of Monetary Economics*, 84, pp. 66–83.

Gollier, C. and Weitzman, M. L. (2010). How should the distant future be discounted when discount rates are uncertain? *Economics Letters*, 107, pp. 350–353.

Gourinchas, P.-O. and Parker, J. A. (2004). Consumption over the life cycle. *Econometrica*, 70(1), pp. 47–89.

Greenstone, M., Kopits, E., and Wolverton, A. (2011). Estimating the Social Cost of Carbon for use in U.S. federal rulemakings: A summary and interpretation. NBER Working Paper 16913.

Groom, B., Hepburn, C., Koundouri, P., and Pearce, D. (2005). Declining discount rates: The long and the short of it. *Environmental & Resource Economics*, 32, pp. 445–493.

Groom, B., Koundouri, P., Panopoulou, E., and Pantelidis, T. (2007). Discounting the distant future: How much does model selection affect the certainty equivalent rate? *Journal of Applied Econometrics*, 22, pp. 641–656.

Hahn, J. and Newey, W. (2004). Jackknife and analytical bias reduction for nonlinear panel models. *Econometrica*, 72(4), pp. 1295–1319.

Hausman, J. (1979). Individual discount rates and the purchase and utilization of energy-using durables. *Bell Journal of Economics*, 10(1), pp. 33–54.

He, J., Liu, H., Sing, T. F., Song, C. and Wong, W.-K. (2020). Superstition, conspicuous spending, and the housing market: Evidence from Singapore. *Management Science*, 66(2), pp. 783–804.

Hoel, M. and Sterner, T. (2007). Discounting and relative prices. *Climatic Change*, 84, pp. 265–280.

HM Treasury (2003). *The Green Book: Appraisal and Evaluation in Central Government*. HM Treasury, London.

HM Treasury (2008). *Intergenerational Wealth Transfers and Social Discounting: Supplementary Green Book Guidance*. HM Treasury, London.

Interagency Working Group on Social Cost of Carbon, IWG (2010, February). *Social Cost of Carbon for Regulatory Impact Analysis under Executive Order 12866.*

Lebègue, D. (2005). *Révision du Taux d'Actualisation des Investissements Publics.* Rapport du Groupe d'Experts, Commisariat général du Plan.

Lettau, M., and Wachter J. A. (2007). Why is long-horizon equity less risky? A duration-based explanation of the value premium. *Journal of Finance*, 62(1), pp. 55–92.

Li, Q. and Pizer, W. A. (2019). The Discount Rate for Public Policy over the Distant Future. NBER Working Paper 25413.

Lornie, J. (1921). Land tenure. In Makepeace, W., Brooke, G. E., and Braddell, R. S. J., editors, *One hundred years of Singapore*, vol. 1, ch. V, pp. 301–314. Oxford University Press, London.

Moore, M. J. and Viscusi, W. K. (1989). The quantity-adjusted value of life. *Economic Inquiry*, 26(3), pp. 369–388.

Moore, M. J. and Viscusi, W. K. (1990). Models for estimating discount rates for long-term health risks using labor market data. *Journal of Risk and Uncertainty*, 3, pp. 381–401.

NAS (2017). *Valuing Climate Damages: Updating Estimation of the Social Cost of Carbon Dioxide, Highlights.* National Academies of Sciences, Engineering, and Medicine, Washington, DC.

Newell, R. G. and Pizer, W. A. (2003). Discounting the distant future: how much do uncertain rates increase valuations? *Journal of Environmental Economics and Management*, 46, pp. 52–71.

Nordhaus, W. D. (1994). *Managing the Global Commons: The Economics of Climate Change.* MIT Press, Cambridge, MA.

Nordhaus, W. D. (2007a). A review of *The Stern Review on the Economics of Climate Change. Journal of Economic Literature*, 45(3), pp. 686–702.

Nordhaus, W. D. (2007b). Critical assumptions in the Stern Review on the Economics of Climate Change. *Science*, 317(5835), pp. 201–202.

Office of Management and Budget, OMB (2003, September). *Circular A-4: Regulatory Analysis.*

Phang, S. Y. (2001). Housing policy, wealth formation and the Singapore economy. *Housing Studies*, 16(4), pp. 443–459.

Phang, S. Y. and Kim, K. (2011). Singapore's housing policies: 1960-2013. In *Frontiers in Development Policy: Innovative Development Case Studies 123.* KDI School and World Bank Institute.

Phillips, P. C. B. and Jin, S. (2015). Business cycles, trend elimination, and the HP filter. Cowles Foundation Discussion Paper No. 2005.

Quinet, E. (2013). *L'évaluation Socioéconomique des Investissements Publics.* Tome 1 Rapport final. Commissariat Général à la Stratégie et à la Prospective.

Stern Review (2007). *The Economics of Climate Change: The Stern Review*. Cambridge University Press, Cambridge.

Stern, N. and Taylor, C. (2007). Climate change: Risk, ethics, and the Stern Review. *Science*, 317(5835), pp. 203–204.

Stern, N. (2013). The structure of economic modeling of the potential impacts of climate change: Grafting gross underestimation of risk onto already narrow science models. *Journal of Economic Literature*, 51(3), pp. 838–859.

Sterner, T. and Persson, U. M. (2008). An even Sterner Review: Introducing relative prices into the discounting debate. *Review of Environmental Economics and Policy*, 2(1), pp. 61–76.

Taylor Wessing. (2012). *Real Estate Finance in Singapore, Part 1 - Land Law in Singapore*. Taylor Wessing.

Train, K. (1985). Discount rates in consumers' energy-related decisions: A review of the literature. *Energy*, 10(12), pp. 1243–1253.

van Binsbergen, J., Brandt, M., and Koijen, R. (2012). On the timing and pricing of dividends. *American Economic Review*, 102(4), pp. 1596–1618.

van Binsbergen, J., Hueskes, W., Koijen, R. S., and Vrugt, E. B. (2014). Equity yields. *Journal of Financial Economics*, 110(3), pp. 503–519.

van Binsbergen, J. H. and Koijen, R. S. J. (2017). The term structure of returns: Facts and theory. *Journal of Financial Economics*, 124(1), pp. 1–21.

Viscusi, W. K. and Moore, M. J (1989). Rates of time preference and valuation of the duration of life. *Journal of Public Economics*, 38(3), pp. 297–317.

Warner, J. T. and Pleeter, S. (2001). The personal discount rate: Evidence from military downsizing programs. *American Economic Review*, 91(1), pp. 33–53.

Weitzman, M. L. (1998). Why the far-distant future should be discounted at its lowest possible rate. *Journal of Environmental Economics and Management*, 36(3), pp. 201–208.

Weitzman, M. L. (2001). Gamma discounting. *American Economic Review*, 91(1), pp. 260–271.

Weitzman, M. L. (2007a). A review of *The Stern Review on the Economics of Climate Change*. *Journal of Economic Literature*, 45(3), pp. 703–724.

Weitzman, M. L. (2007b). Subjective expectations and asset return puzzles. *American Economic Review*, 97(4), pp. 1102–1130.

Weitzman, M. L. (2013). Tail-hedge discounting and the social cost of carbon. *Journal of Economic Literature*, 51(3), pp. 873–882.

Wong, S. K., Chau, K. W., Yiu, C. Y., and Yu, M. K. W. (2008). Intergenerational discounting: A case from Hong Kong. *Habitat International*, 32, pp. 283–292.

Zeldes, S. P. (1989). Consumption and liquidity constraints: An empirical investigation. *Journal of Political Economy*, 97(2), pp. 305–346.

Table 1: Descriptive statistics for transactions of new privately developed apartments.

Panel I: Sales volume over time and by lease years remaining	Year new apartment was sold:				Total
	1995-99	2000-04	2005-09	2010-15	(# of units)
	Full sample: All areas				
Perpetual leases	13,599	10,989	27,206	21,040	72,834
Multi-century leases					
876 to 986 years	3,093	825	353	0	4,271
825 to 875 years	657	330	2,524	1,372	4,883
Multi-decade leases					
91 to 99 years	18,932	16,207	15,630	45,336	96,105
87 to 90 years	0	44	9	835	888
56 to 63 years	0	0	14	223	237
Total (# of units)	36,281	28,395	45,736	68,806	179,218
	Areas w/ sales of at least multi-century and perpetual leases				
Perpetual leases	5,384	2,583	6,775	4,417	19,159
Multi-century leases					
876 to 986 years	1,931	763	347	0	3,041
825 to 875 years	388	272	2,147	1,242	4,049
Multi-decade leases					
91 to 99 years	1,125	935	536	2,192	4,788
87 to 90 years	0	33	2	0	35
56 to 63 years	0	0	0	0	0
Total (# of units)	8,828	4,586	9,807	7,851	31,072
Panel II: Other transaction variables	N	Mean	Std. Dev.	Minimum	Maximum
Transaction price (S$)	179,218	1,336,314	1,181,158	308,248	43,396,372
Apartment size (m^2)	179,218	107.999	47.440	24	1,186
Price per m^2 (S$)	179,218	12,374	5,530	2,670	78,068
Apartment story	179,218	8.999	7.590	1	70
On 1st story (1=yes)	179,218	0.059	0.236	0	1
On top story (1=yes)	179,218	0.086	0.280	0	1
Project size (# of units)	179,218	367.563	266.034	1	1,371
Large land parcel (1=yes)	179,218	0.703	0.457	0	1
Sold $\geq$ 1 year after complete	179,218	0.035	0.185	0	1
Sold during construction	179,218	0.869	0.337	0	1
Distance to the nearest shopping mall (km)	179,218	0.949	0.654	0	3.098
Project includes a swimming pool (1=yes)	177,689	0.754	0.431	0	1
Project includes a gym (1=yes)	177,689	0.770	0.421	0	1
Project includes a tennis court (1=yes)	177,689	0.598	0.490	0	1
Buyer age (years)	54,548	42.144	9.526	18	93

Notes: Purchases of new privately developed apartments from 1995 to 2015. Statistics are shown for the full sample of transactions across areas ($N = 179,218$) and for a subsample in more homogeneous areas, defined as 3-digit areas in which at least multi-century and perpetual leases were traded in sample ($N = 31,072$). In panel I, we group the joint density into cells of similar remaining lease only for ease of exposition. Prices in S$ (base CPI January 2014). Apartments on the 1st story may be next door to common areas. Apartments on the top story tend to have a larger recorded size, including less valuable external balcony space that depresses the price per m^2. Project size is the number of apartments transacted within the condominium project, for which caveats were lodged with the Singapore Land Authority (Appendix A.1). Large land parcel is a project-specific dummy variable as defined by the Urban Redevelopment Authority, namely a parcel at least 0.4 hectare in land area. We observe buyer age for a subset of transactions (taking the average when there is more than one buyer, e.g., two spouses).

Table 2: Discount rate as a smooth function of time.

Function of time:	r_t exponential		r_t logarithmic		r_t hyperbolic	
Location controls:	5-digit	3-digit	5-digit	3-digit	5-digit	3-digit
Dependent variable: Log price	(1a)	(2a)	(3a)	(4a)	(5a)	(6a)
Slope parameter, γ_2	-0.0046	-0.0046	-0.0102	-0.0097	-0.1537	-0.1430
	(0.0001)	(0.0001)	(0.0005)	(0.0004)	(0.0119)	(0.0078)
Avg. $\hat{\phi}_i$ 825-986y/avg. $\hat{\phi}_i$ perp.	0.94	0.94	1.00	1.00	1.00	1.00
Avg. $\hat{\phi}_i$ 87-99y/avg. $\hat{\phi}_i$ perpetual	0.86	0.87	0.85	0.87	0.85	0.87
Jackknife estimator	-0.0046	-0.0046	-0.0103	-0.0100	-0.1535	-0.1492
Model cross-valid. Mean Squared Error ($\times 10^6$ S$/m^2 squared)	2.1205	6.8195	2.1184	6.8392	2.1184	6.8397
Dependent var.: Price per m^2	(1b)	(2b)	(3b)	(4b)	(5b)	(6b)
Slope parameter, γ_2	-0.0048	-0.0046	-0.0111	-0.0095	-0.1750	-0.1380
	(0.0003)	(0.0003)	(0.0010)	(0.0016)	(0.0258)	(0.0264)
Avg. $\hat{\phi}_i$ 825-986y/avg. $\hat{\phi}_i$ perp.	0.91	0.94	1.00	1.00	1.00	1.00
Avg. $\hat{\phi}_i$ 87-99y/avg. $\hat{\phi}_i$ perpetual	0.83	0.86	0.82	0.87	0.82	0.87
Jackknife estimator	-0.0047	-0.0052	-0.0110	-0.0132	-0.1595	-0.1974
Model cross-valid. Mean Squared Error ($\times 10^6$ S$/m^2 squared)	2.2966	6.9002	2.2932	6.9133	2.2931	6.9132

Notes: This table reports the slope parameter, γ_2, when the discount rate is a smooth parametric function of time under 12 alternative specifications. Standard errors, in parentheses, are clustered by building. In the top panel (columns 1a-6a), the dependent variable is the natural logarithm (log) of the apartment's transaction price per square meter of floor area. In the bottom panel (columns 1b-6b), the dependent variable is the apartment's transaction price per square meter of floor area (S$/m^2). Across columns, the discount rate is a smooth exponential, logarithmic, or hyperbolic function of time, and we further vary the granularity of location controls. With 5-digit controls, 96% of 1516 locations (e.g., 12772x) contain a single lease type (multi-decade, multi-century or perpetual). With 3-digit controls, a lower 42% of 187 locations (e.g., 127xxx) contain a single lease type. All specifications are estimated by NLLS on the full sample of 179,218 new apartment purchases from 1995 to 2015 and, besides location, include as housing utility shifters and market effects: apartment size and its square; apartment story and its square; an indicator for apartment on 1st story; an indicator for apartment on top story (and interactions with apartment size and its square); project size and its square; a project-specific indicator for a large land parcel; purchase-to-completion bins of width 1 year; year-of-purchase fixed effects; and quarter-of-year fixed effects. Solver Knitro using the interior-point algorithm with the year-1 discount rate γ_1 fixed at 4% p.a. and the lower bound to the discount rate γ_3 fixed at 0.01% p.a. Estimates are robust to using optimization with a global search algorithm. See the text for the jackknife adjustment. For model cross validation (Appendix A.4), we randomly partition the data (within 5-digit or 3-digit location) into 10 folds; we take 9 folds as the training set and 1 fold as the test set, repeating ten times as we shift the test set to the next fold. We report the Mean Squared Error over observations in all 10 test sets.

Table 3: Discount rate restricted to be flat over time.

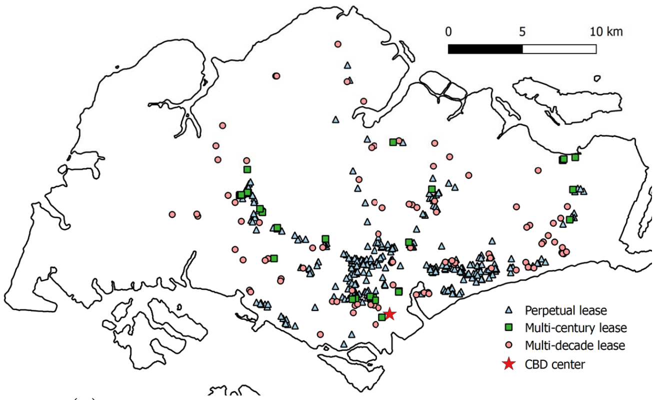
Function of time:	r constant		Jump at $t = 100$		Jump at $t = 800$	
Location controls:	5-digit (1)	3-digit (2)	5-digit (3)	3-digit (4)	5-digit (5)	3-digit (6)
Dependent variable: Log price						
Discount rate (p.a.)	0.0205	0.0215				
	(0.0011)	(0.0008)				
Disc. rate, up to year 100 (p.a.)			0.0276	0.0284		
			(0.0079)	(0.0057)		
Disc. rate, year 100 on (p.a.)			0.0121	0.0128		
			(0.0074)	(0.0059)		
Disc. rate, up to year 800 (p.a.)					0.0205	0.0240
					(0.0011)	(0.0020)
Disc. rate, year 800 on (p.a.)					0.0059	0.0000
					(0.0010)	(0.0000)
Jump in rates at year 100 or 800 (p.a.)			-0.0155	-0.0157	-0.0146	-0.0240
			(0.0152)	(0.0116)	(0.0022)	(0.0020)
Avg. $\hat{\phi}_i$ 825-986y / avg. $\hat{\phi}_i$ perp.	1.00	1.00	1.00	1.00	1.00	0.96
Avg. $\hat{\phi}_i$ 87-99y/avg. $\hat{\phi}_i$ perpetual	0.85	0.87	0.85	0.87	0.85	0.86
Model cross-valid. Mean Squared Error ($\times 10^6$ S\$/m^2 squared)	2.1185	6.8413	2.1127	6.8366	2.1185	6.8407

Notes: This table reports constant discount rates under 6 alternative specifications. Standard errors, in parentheses, are clustered by building. The dependent variable is the log of the apartment's transaction price per square meter of floor area. Across columns, we allow jumps at $t = 100$ or $t = 800$, and we further vary the granularity of location controls. Other housing utility shifters and market effects as in Table 2. All specifications are estimated by NLLS on the full sample of 179,218 new apartment purchases from 1995 to 2015. Solver Knitro using the interior-point algorithm with the discount rate $r = \gamma$ (or discount rates $r_1 = \gamma_1$ and $r_2 = \gamma_2$) constrained between 0 and 10% p.a. (i.e., 0.1). For model cross validation (Appendix A.4), we randomly partition the data (within 5-digit or 3-digit location) into 10 folds; we take 9 folds as the training set and 1 fold as the test set, repeating ten times as we shift the test set to the next fold. We report the Mean Squared Error over observations in all 10 test sets.

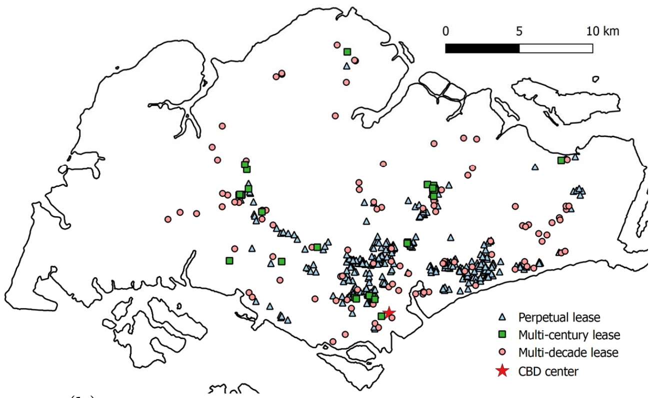
Table 4: Discount rate as a step function of time with a rate acceleration penalty.

Function of time:	10 steps		5 steps	
Location controls:	5-digit (1)	3-digit (2)	5-digit (3)	3-digit (4)
Dependent variable: Log price				
Discount rate, year 1 to 100 (p.a.)	0.0248	0.0349	0.0247	0.0320
	(0.0011)	(0.0010)	(0.0008)	(0.0001)
Discount rate, year 101 to 200 (p.a.)	0.0173	0.0195	0.0172	0.0196
	(0.0008)	(0.0007)	(0.0008)	(0.0001)
Discount rate, year 201 to 300 (p.a.)	0.0114	0.0058	0.0114	0.0081
	(0.0011)	(0.0005)	(0.0008)	(0.0001)
Discount rate, year 301 to 400 (p.a.)	0.0075	0.0000	0.0077	0.0017
	(0.0018)	(0.0003)	(0.0008)	(0.0001)
Disc. rate, year 401 to 500, or year 401 on (p.a.)	0.0053	0.0000	0.0059	0.0012
	(0.0026)	(0.0002)	(0.0008)	(0.0001)
Discount rate, year 501 to 600 (p.a.)	0.0044	0.0000		
	(0.0034)	(0.0002)		
Discount rate, year 601 to 700 (p.a.)	0.0045	0.0000		
	(0.0042)	(0.0004)		
Discount rate, year 701 to 800 (p.a.)	0.0052	0.0000		
	(0.0050)	(0.0006)		
Discount rate, year 801 to 900 (p.a.)	0.0062	0.0000		
	(0.0058)	(0.0009)		
Discount rate, year 901 on (p.a.)	0.0073	0.0028		
	(0.0067)	(0.0011)		
Average $\hat{\phi}_i$ 825-986y / average $\hat{\phi}_i$ perpetual	1.00	0.97	1.00	0.97
Average $\hat{\phi}_i$ 87-99y / average $\hat{\phi}_i$ perpetual	0.85	0.86	0.85	0.86
Objective function components (at fitted parameters):				
Smoothing parameter, λ (fixed)	3027	3027	3027	3027
Sum of squared second differences (SSSD)	0.000013	0.000106	0.000014	0.000061
Rate acceleration penalty, $\lambda\times$SSSD	0.0382	0.3197	0.0438	0.1856
Residual sum of squares (RSS)	1513.43	3712.40	1513.43	3713.41
Objective = RSS+$\lambda\times$SSSD	1513.47	3712.72	1513.47	3713.60
Model cross-validation Mean Squared Error ($\times 10^6$ S\$/m^2 squared)	2.1183	6.8189	2.1183	6.8225

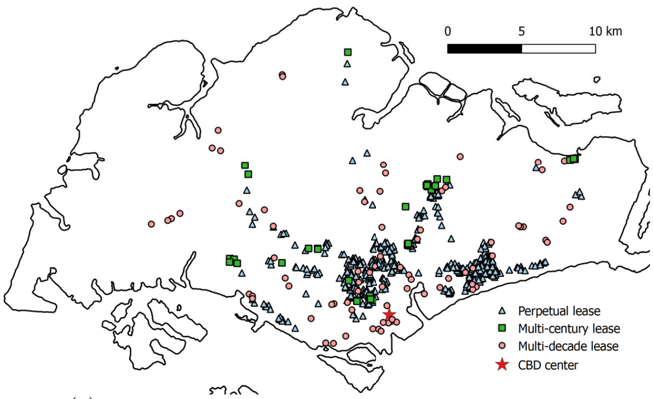
Notes: This table shows discount rates as a step function of time under 4 alternative specifications. Standard errors, in parentheses, are clustered by building. The dependent variable is the log of the apartment's transaction price per square meter of floor area. Across columns, we allow jumps at steps one century wide until the fifth or tenth century, and we further vary the granularity of location controls. Other housing utility shifters and market effects as in Table 2. All specifications are estimated by penalized NLLS on the full sample of 179,218 new apartment purchases from 1995 to 2015. Solver Knitro using the interior-point algorithm with discount rates r_1 to r_{10} constrained between 0 and 10% p.a. (i.e., 0.1). As explained, the objective function adds to the RSS a rate acceleration penalty given by a smoothing parameter λ times the sum of squared second differences, i.e., $((r_3 - r_2) - (r_2 - r_1))^2 + (r_4 - r_3) - (r_3 - r_2))^2 + ...$ where $r_1, r_2, r_3, ...$ denote annual discount rates from year 1 to 100, year 101 to 200, year 201 to 300,... We fix $\lambda = 3027$, which is twice the no-penalty RSS for a specification with 5-digit controls (see Appendix Table A.11 and Figure A.7 for other λ). As shown, the rate acceleration penalty accounts for a small fraction of the optimized objective function, e.g., 0.04 out of 1513.43 in column 3. For model cross validation (Appendix A.4), we randomly partition the data (within 5-digit or 3-digit location) into 10 folds; we take 9 folds as the training set and 1 fold as the test set, repeating ten times as we shift the test set to the next fold. We report the Mean Squared Error over observations in all 10 test sets.



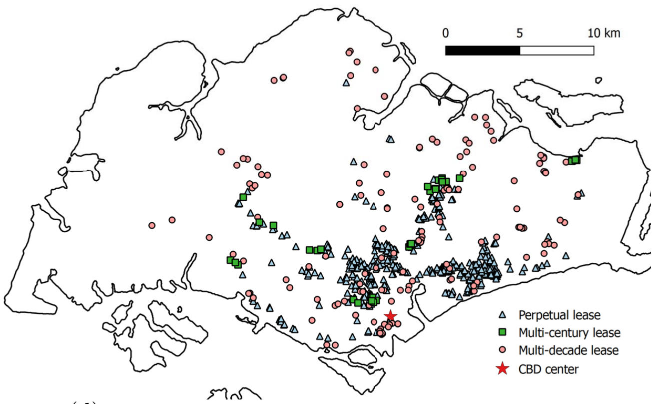
(a) Projects with new apartments sold in 1995-1999



(b) Projects with new apartments sold in 2000-2004

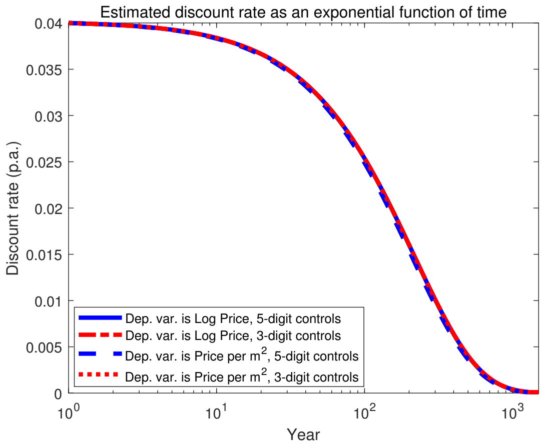


(c) Projects with new apartments sold in 2005-2009

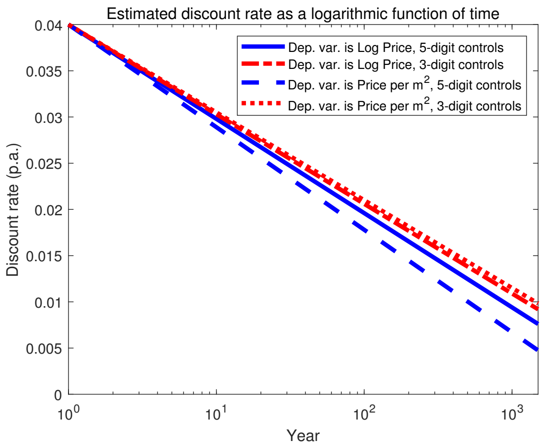


(d) Projects with new apartments sold in 2010-2015

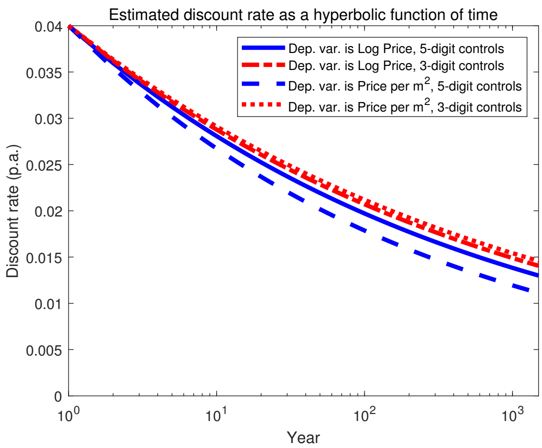
Figure 1: Condominium project location by lease type and period of sale. CBD marks the Central Business District, centered at One Raffles Place.



(a) r_t as an exponential function of time

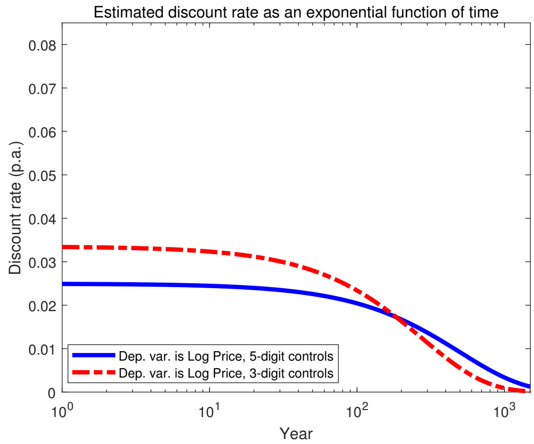


(b) r_t as a logarithmic function of time

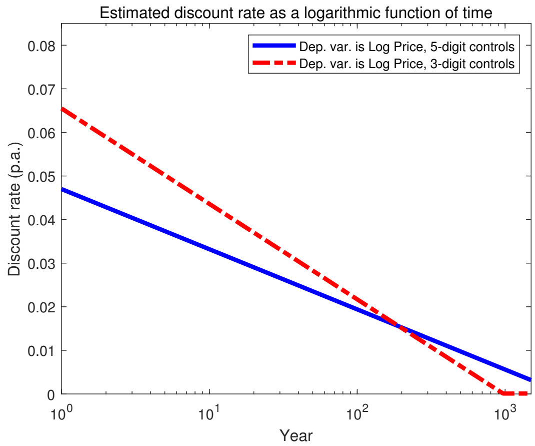


(c) r_t as a hyperbolic function of time

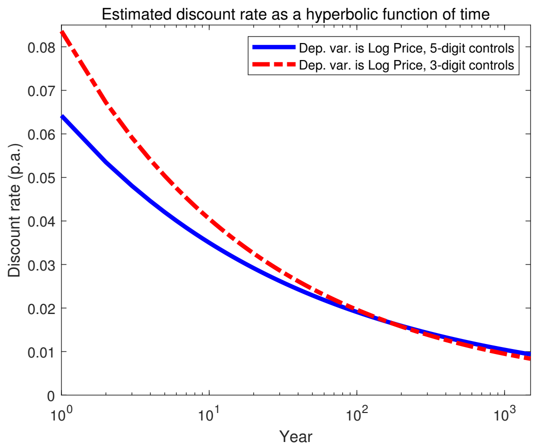
Figure 2: Estimated discount rate as a smooth (a) exponential, (b) logarithmic, and (c) hyperbolic function of time, plotted on a log time scale. The discount rate r_t discounts benefits from year $t+1$ to year t. Source: Table 2 estimates based on the full sample, varying the model and location fixed effects (within panel), with the year-1 discount rate γ_1 and the lower bound to the discount rate γ_3 fixed at 4% p.a. and 0.01% p.a., respectively.



(a) r_t as an exponential function of time

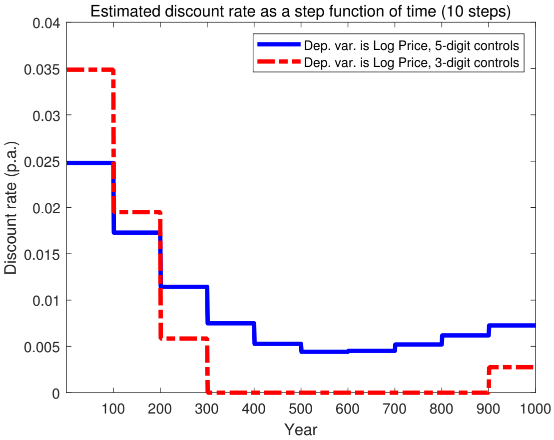


(b) r_t as a logarithmic function of time

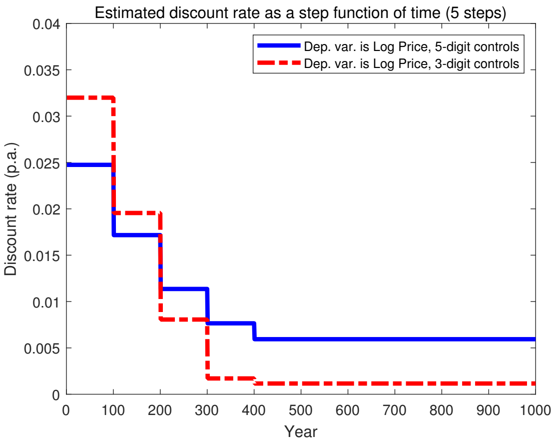


(c) r_t as a hyperbolic function of time

Figure 3: Estimated discount rate as a smooth (a) exponential, (b) logarithmic, and (c) hyperbolic function of time, plotted on a log time scale. Instead of fixing the year-1 discount rate γ_1 at 4% p.a., here we estimate this parameter (along with the slope parameter) subject to the constraint that it lies between 0.01% and 10%. The discount rate r_t discounts benefits from year $t+1$ to year t. Source: Appendix Table A.5 estimates based on the full sample, varying the location fixed effects (within panel), with the lower bound to the discount rate γ_3 fixed at 0.01% p.a.

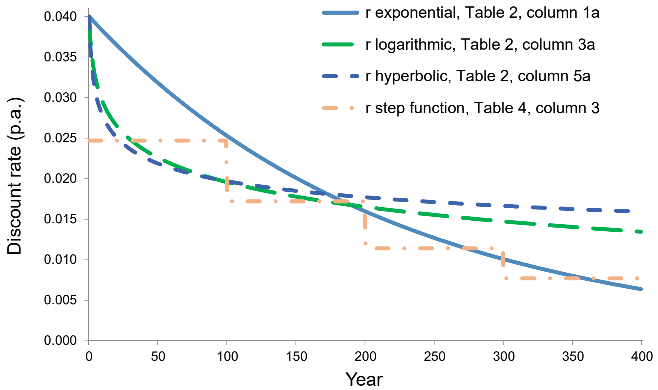


(a) r_t as a step function of time (10 steps)

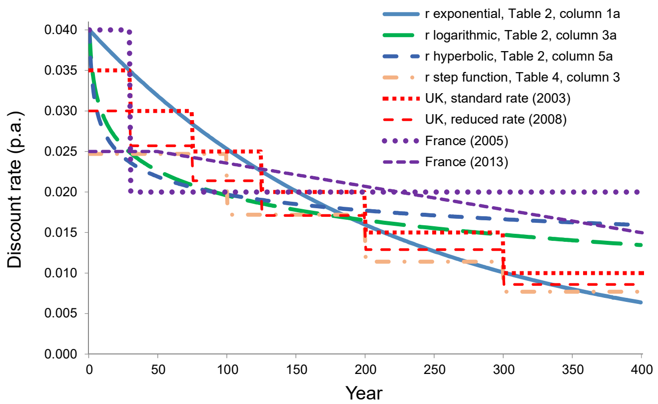


(b) r_t as a step function of time (5 steps)

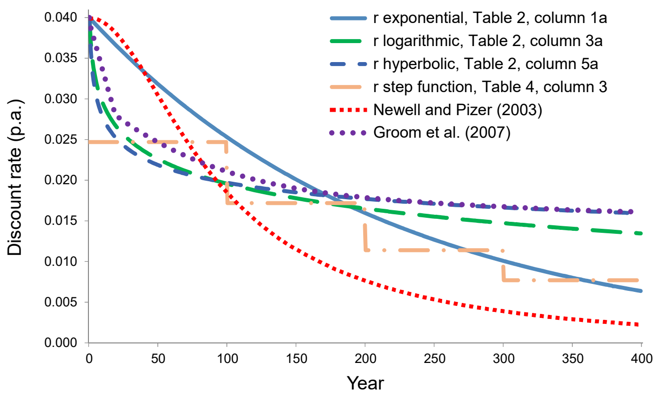
Figure 4: Estimated discount rate as a step function of time with a rate acceleration penalty similar in spirit to the HP filter, plotted on a linear time scale. We allow jumps at steps one century wide until (a) the fifth century, and (b) the tenth century, with a constant discount rate thereafter. The discount rate r_t discounts benefits from year $t+1$ to year t. Source: Table 4 estimates based on the full sample, varying the location fixed effects (within panel).



(a) Estimated discount rate schedules



(b) Comparison to UK and France discounting policy schedules



(c) Comparison to Newell and Pizer (2003) & Groom et al. (2007)

Figure 5: Estimated DDR based on (a) specifications in Tables 2 and 4, and compared to discount rate schedules ("forward rates") (b) used by UK and French governments to evaluate policy (HM Treasury, 2003, 2008; Lebègue, 2005; Quinet, 2013), and (c) estimated by Newell and Pizer (2003) (autoregressive random walk) and Groom et al. (2007) (AR process with time-dependent parameters). We specify location controls at the tight 5-digit level, but findings are similar with less granular fixed effects. The discount rate r_t discounts benefits from year $t+1$ to year t. Time in linear scale. We thank Richard Newell and Billy Pizer for sending us their simulated discount factors, from which we computed the plotted discount rate schedule. Groom et al. discount rates were obtained from an earlier 2004 working paper (Table 2), that listed certainty-equivalent discount rates for an illustrative grid of horizons, to which we fitted smooth polynomials for the purpose of plotting the rate schedule. We draw the revised French schedule's "gradual decline to 1.5 percent in the more distant future" as linear through year 400.